Enrollment No: ______________________ **Exam Seat No.:**____________________

C. U. SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Topology

Subject Code: 5SC01TOP1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 05/01/2023** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Define: Topology **(02)**

b. Is it true that $(A \cup B)^{\circ} \neq A^{\circ} \cup B^{\circ}$? Justify your answer. **(02)**

c. Define: Lower limit topology. **(01)**

d. $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b, c\}\}$and $A = \{a, c\}$then find A°. **(02)**

Q-2 **Attempt all questions** **(14)**

a. If $X = \{a, b, c\}$, and $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ then prove that (X, τ) is topological space. Also find all the τ- closed subset of X. **(06)**

b. Let (X, τ) is topological space then show that any finite union of closed sets is closed. **(05)**

c. Let (R, τ_u) is topological space. And $A = (0,1)$ then find $A', \bar{A}, and\ BdA$. **(03)**

OR

Q-2 **Attempt all questions** **(14)**

a. State and prove Co-finite topology **(05)**

b. Let Y be a subspace of X then prove that a set A is closed in Y iff $A = Y \cap C$. Where C is closed in X. **(07)**

c. Define: Door Space with example. **(02)**

Q-3 **Attempt all questions** **(14)**

a. State and prove Subspace Topology. **(05)**

b. Let (X, τ) be a topological space. and A, B be two subsets of X then prove that **(04)**

(i) $X^{\circ} = X,\ \phi^{\circ} = \phi$

(ii) If $A \subset B$ then $A^{\circ} \subset B^{\circ}$

(iii) $(A \cap B)^{\circ} = A^{\circ} \cap B^{\circ}$

c. Let X be a topological space and β be the basis for the set X. Define $\tau = \{U / each\ x \in U\ \exists\ B \in \beta\ such\ that\ x \in B \subset U\}$ then Prove that τ is topology on X. **(05)**

OR

Q-3 **Attempt all questions** **(14)**

a. Let (X, τ) be a topological space. And let $A \subseteq X$. Then prove that $\bar{A} = A \cup A'$. **(06)**

b. Let (X, τ) be a topological space and $A, B \subset X$ then show that **(06)**

(i) $Bd(A \cup B) \subset BdA \cup BdB$

(ii) $Bd(A \cap B) \subset BdA \cup BdB$

c. Let (X, τ) be a topological space and $A, B \subset X$ then show that $ext(A \cup B) = ext\ A \cap ext\ B$ **(02)**

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a. Define: Continuous Function. **(02)**

b. State Pasting Lemma. **(02)**

c. Define Compact space. **(02)**

d. True or false: Every $T_1 -$ space is $T_2 -$ space. **(01)**

Q-5 **Attempt all questions** **(14)**

a. Let $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, and $Y = \{1,2,3\}, \tau_2 = \{\phi, Y, \{2\}, \{1,3\}\}$. Is $f: X \to Y$ Homeomorphism. Define by $f(a) = 2, f(b) = 3, f(c) = 1$. **(05)**

b. Prove that Homeomorphism is an equivalence relation. **(06)**

c. Show that Projection map are continuous. **(03)**

OR

Q-5 **Attempt all questions** **(14)**

a. Prove that continuous image of compact set is compact. **(07)**

b. Prove that every closed subset of compact space is closed. **(05)**

c. Define T_4 Space with example. **(02)**

Q-6 **Attempt all questions** **(14)**

a. Prove that every compact subset of T_2- space is closed. **(07)**

b. State and prove Sequence lemma. **(07)**

OR

Q-6 **Attempt all Questions** **(14)**

a. Prove that a topological space (X, τ) is T_1- space iff $\{x\}$ is closed for $\forall x \in X$. **(05)**

b. Let (X, τ_1) and (Y, τ_2) be topological space. If $f: X \to Y$ be continuous function then so that $x_n \to x \Rightarrow f(x_n) \dashrightarrow f(x)$ and converse hold if X is Metrizable. **(05)**

c. Let (X, τ) is a topological space and $A \subset X$, then $x \notin \bar{A}$ iff there exist an open set U containing x that does not intersect A. **(04)**